

# Coevolution of Glauber-like Ising dynamics on typical networks

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We consider coevolution of site status and link structures from two different initial networks: a one dimensional Ising chain and a scale free network. The dynamics is governed by a preassigned stability parameter  $S$ , and a rewiring factor  $\phi$ , that determines whether the Ising spin at the chosen site flips or whether the node gets rewired to another node in the system. This dynamics has also been studied with Ising spins distributed randomly among nodes which lie on a network with preferential attachment. We have observed the steady state average stability and magnetisation for both kinds of systems to have an idea about the effect of initial network topology. Although the average stability shows almost similar behaviour, the magnetisation depends on the initial condition we start from. Apart from the local dynamics, the global effect on the dynamics has also been studied. These parameters show interesting variations for different values of  $S$  and  $\phi$ , which helps in determining the steady-state condition for a given substrate.

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## I. INTRODUCTION

Statistical mechanics and network theory helps us to describe and also analyse the collective features of large systems such as human societies, by studying macroscopic parameters, without the knowledge of the microscopic (read individual) details. Complex web-like structures describe a wide variety of systems of high technological and intellectual importance. The statistical properties of many such networks are being studied recently with much interests. Such networks, with complex topology are common in nature and examples include the world wide web, the Internet structure, social networks, communication networks, neural networks to name a few [1–3].

In social networks, most of the individuals interact with a limited number of fellow persons and this number is almost negligible compared to the total number of individuals comprising the network. In spite of this, human societies exhibit fascinating global features [4]. One social phenomena that is being widely explored by Physicists in recent times is opinion formation or opinion dynamics. Although individual opinions in a society or group might vary, however after undergoing a particular dynamics, the group tends to present a single opinion. When the individuals are all differing in their respective opinions, the system is heterogeneous and a Physicist would call it a ‘disordered system’; dynamical interaction would make individuals having same or similar opinion

to get linked and those having dissimilar ideas to get detached from each other. It might also happen that an influential individual succeeds in altering the point of view of another individual in the group. When, after undergoing such dynamics, a consensus, or agreement is reached, the system would be acclaimed by a Physicist as ‘ordered’ [5].

One of the key facts to be kept in mind while designing or simulating a social network is that in this case while the individual nodes change their states, the network also changes its topology due to the formation or severing of links between pairs of nodes. Hence not only do the nodes evolve, the network as a whole also evolves in time due to change in its link structure. Hence a correct representation of social dynamics should include a “coevolution” of state dynamics and network topology [6]. This class of models, where such coevolution has been studied have been published profusely in past years [7].

In the present paper, we study such a system where a coevolution of node status and the link structure of the network takes place. Since binary opinion holds a major place in the opinion dynamics literature, we represent the agents by nodes and their opinions by spins that can be either plus or minus. We study different network topologies on which these spins are placed either randomly or in an antiferromagnetic fashion. We study a one dimensional Ising chain as well as a network with preferential attachment [1]. We study different features such as the average stability, magnetisation, and number of free nodes. We have analysed the system from the transformation patterns of the above parameters.

## II. THE SYSTEMS AND THE DYNAMICS

We have studied mainly two kinds of Ising systems as our starting point: (i) a one dimensional Ising chain

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with nearest neighbour interaction and (ii) a network grown by preferential attachment scheme, the sites of which are assigned by spins  $\sigma = \pm 1$  with probability  $1/2$ . The nodes here are the individuals and the spin states here represent individual opinion is which considered to be binary (e.g., *yes* or *no*) The dynamics follow the update rule as described below [6]. The chief parameter deciding the update rule is the *stability factor* which is defined as follows:

**Stability factor** : The stability factor  $s_i$  of a particular node  $i$  is defined as the ratio  $l_i/k_i$ , where  $k_i$  is the number of links arising from the node, i.e., its degree and  $l_i$  gives the number of neighbours having the same sign as the  $i^{th}$  node.

Once the lattice is generated upto a desired size, say,  $N$ , and each node has a particular value of spin,  $l_i$ ,  $k_i$  and  $s_i$ , we apply the update rule as follows:

(a) any  $i^{th}$  node is selected randomly and its  $s_i$  calculated.

(b) We denote the node to be stable if  $s_i \geq S$ , where  $S$  is a preassigned value that we call “target stability value” and  $0 \leq S \leq 1$ . In this case the node does not change its sign nor does it rewire. On the other hand, if  $s_i < S$ , then a neighbour  $j$  of  $i$  is chosen randomly, such that  $\sigma_i \neq \sigma_j$  and

(i) with a preassigned probability  $\phi$  the node  $i$  severs its link with  $j$  and attaches with another node  $l$  which is chosen at random from the rest of the network so that  $\sigma_i = \sigma_l$ ; provided  $j$  and  $l$  were not connected previously  
(ii) with a probability  $(1 - \phi)$  the node  $i$  flips its spin.

It is worth mentioning that if during the rewiring process, any node gets temporarily disconnected from the network, its stability factor is assigned as 1, i.e.,  $s_i = 1$  for such a node. This implies that a free node is stable and independent of the dynamics going on, until it gets connected during the rewiring of some other node.

The chief tunable parameters here are the preassigned stability  $S$  and the rewiring probability  $\phi$ . The value of  $S$  determines the density of similar signed neighbours required for a node to be stable. The goal of the dynamics obviously is to reach a stable, ordered state starting from an initial random state.

### III. DIFFERENT CASES

#### A. Randomly initialised one dimensional chain

As a starting point, the substrate chosen for the rewiring and/or spin flipping dynamics to take place is a one dimensional chain of  $N$  spins, where we distribute plus and minus (or up and down) spins randomly. Therefore each node has an equal probability of having either  $+\sigma$  or  $-\sigma$ ; in our study,  $|\sigma| = 1$ . Each node is connected only to its two nearest neighbours, i.e., its adjacent nodes, before the updating begins. For each

randomly chosen  $i^{th}$  node, the value of the stability  $s_i$  is determined from the values of  $l_i$  and  $k_i$  and the aforementioned update rule is applied. For this configuration, each node may initially have any one of the following three values of  $s_i$ , viz., 1.0, 0.5 and 0.0. Keeping this in mind, we classify the sites in terms of their  $s_i$  values as:

(i) a site whose stability is 1.0 (connected only with other sites of same spin polarity) is called an *inactive site* or *i-site*. For  $\phi = 0.0$ , i.e., for no rewiring, a site whose two adjacent sites are identical is an *i-site*.

(ii) a site whose stability is between 0.0 and 1.0, is called a *dormant site* or *d-site*, because these sites flip according to the value of  $S$ . For  $\phi = 0.0$ , i.e., for no rewiring a site whose two adjacent sites are mutually opposite is a *d-site* and the stability of such a site is always 0.5.

(iii) a site whose stability is 0.0 (connected only with all other sites of opposite spin polarity), is called an *active site* or *a-site*, because these sites always flip. For  $\phi = 0.0$ , i.e. for no rewiring a site whose two adjacent sites are oppositely oriented to the site itself is an *a-site*.

After allowing the system to reach the equilibrium configuration ( $\sim 1000$  time steps), we measure the following quantities:

- (i) The average stability per node  $\langle s \rangle = \Sigma s_i / N$ .
- (ii) magnetisation  $m = \Sigma_i \sigma_i / N$
- (iii) the fraction of free nodes left  $n_f$

We observe here that the average stability per node decreases with increasing probability of rewiring for  $S \leq 0.5$ , whereas for  $S > 0.5$ , the value of  $\langle s \rangle$  remains almost unaltered ( $\sim 1$ ). For  $S \leq 0.5$ , two branches are obtained which converge to the same value for  $\phi = 0$  (Fig. 1a).

In case of random initialisation with two links for each site, we may assume that if there are total  $N$  spins, there will be  $N/4$  *a* sites,  $N/4$  *i* sites and  $N/2$  *d* sites, contributing a stability of 0.50. Let us consider the dynamics when  $\phi = 0$  and  $S < 0.5$ , i.e., when an *a-site* certainly converts into an *i-site* (Fig.2. Path A). Simultaneously an adjacent *d-site* may convert to an *i-site* (Fig.2. Path B) or an *a-site* convert to a *d-site* (Fig.2. Path C). Initially for random configuration the ratio of *d-site* to *a-site* is 2 : 1 and conversion of each *a-site* leads to conversion of 2 adjacent *d* or *a* sites. So it is expected that conversion of each *a-site* corresponds to the transformation of  $4/3$  *d* sites to  $4/3$  *i* sites and  $2/3$  *a* sites to  $2/3$  *d* sites. So for a single site update,  $5/3$  *a* and  $2/3$  *d* sites vanish and  $7/3$  *i* sites appear. The transformation equations are as follows :

$$a \rightarrow i \quad (\text{Fig2.PathA})$$

$$\frac{4}{3}d \rightarrow \frac{4}{3}i \quad (\text{Fig2.PathB})$$

$$\frac{2}{3}a \rightarrow \frac{2}{3}d \quad (\text{Fig2.PathC})$$

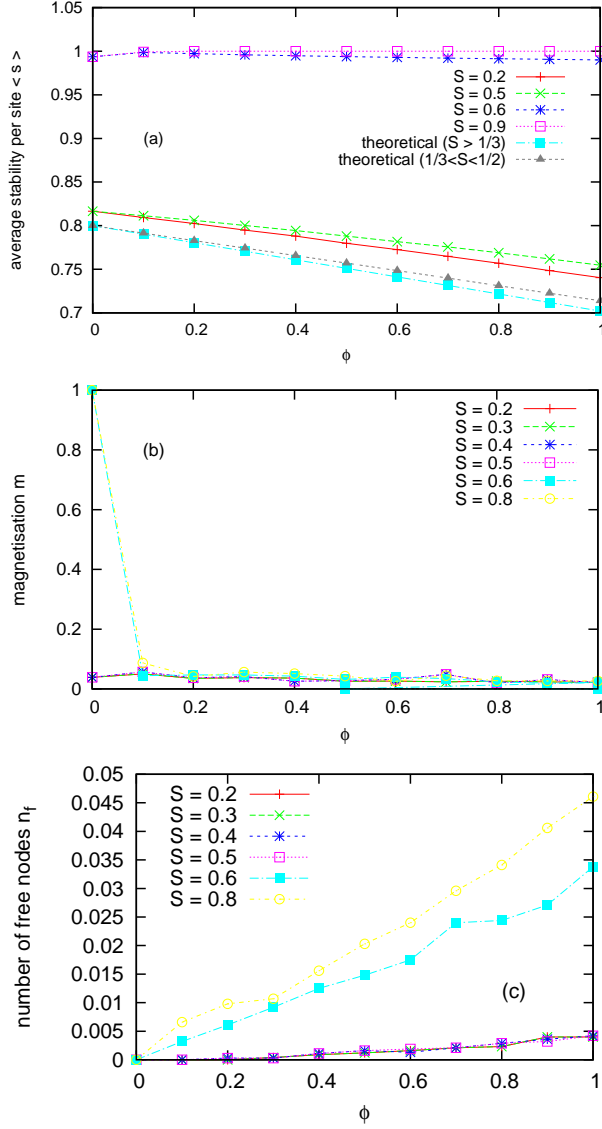


FIG. 1: (a) Average stability per site  $\langle s \rangle$  vs  $\phi$  for different values of  $S$ ; the two bottom plots are the ones obtained from theoretical calculations for  $S < 0.5$  (explanation in text); (b) Magnetisation ( $m$ ) vs  $\phi$ ; (c) Fraction of free nodes remaining in the system ( $n_f$ ) vs  $\phi$  plot for different values of the preassigned stability factor  $S$

$$\frac{5}{3}a + \frac{2}{3}d \rightarrow \frac{7}{3}i \quad (\text{net conversion}) \quad (1)$$

Initially there are  $N/4$   $a$  sites and the dynamics continues until they all disappear. Instead of going into the microscopic details, we assume that the respective sites decay at a constant rate. So it takes  $3N/20$  steps, and during this time  $3N/20 \times 2/3 = N/10$   $d$  sites vanishes. So finally,  $2N/5$   $d$  sites are left and the remaining are  $3N/5$   $i$  sites leading to a stability of 0.8.

Now let us concentrate on the dynamics followed by the system for  $\phi = 1.0$  and  $S < 0.5$ , i.e., when stabil-

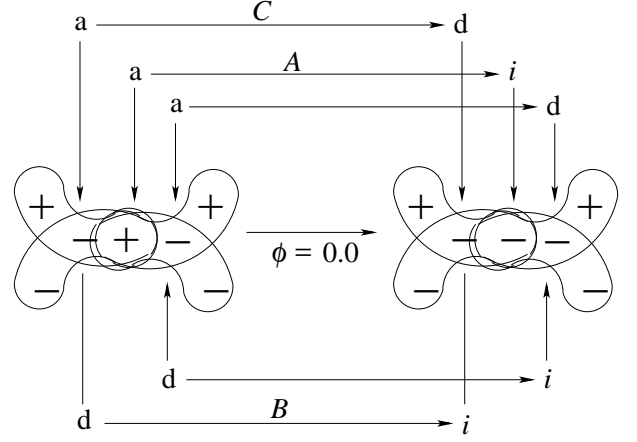


FIG. 2: The above figure describes how a given  $a$  site undergoes transformation when  $S < 0.5$  and  $\phi = 0.0$ . In the figure, an active site ( $+$ ), flips to an inactive site ( $-$ ) while its neighbouring sites, which might be active or dormant, transform to dormant or inactive sites respectively. Consequent change in stability corresponding to the entire update has been calculated to be  $\Delta \langle s \rangle = 0.3$ .

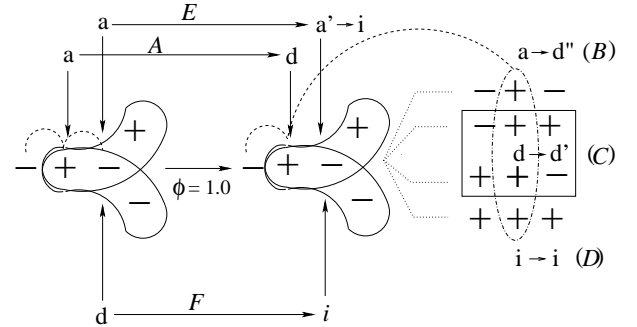


FIG. 3: When  $\phi = 1.0$ , there is no spin flip, but an  $a$  site would sever its link from one of its neighbours and link to a distant site, which might be  $a$ ,  $d$ , or  $i$ . The above figure shows such a transformation and depicts how the configuration of the distant site might change due to addition of a like signed node and the four possibilities are shown. The original neighbour, from whom the link has been severed will also change to an  $a$  or an  $i$  site. In this case, the theoretically calculated value of  $\Delta \langle s \rangle = 0.202$  or  $0.214$  (details in text)

ity increases only through the process of rewiring. In this case also the  $a$ -sites take the major role, while the other sites are affected indirectly. A distinct property that largely discriminates this dynamics from the previously described one, is that during a single update only one of the adjacent sites get affected due to the conversion of the main  $a$ -site and the other one remains totally undisturbed. Instead of that a site at an arbitrary distance from the principal site with which it gets newly connected may give rise to various configurations depending upon the initial state it starts off. Let us go through all the possible changes one by one. The compulsory change is that of an  $a$ -site converting to a  $d$ -site (Fig.3 Path A)

by getting disconnected from any of the adjacent oppositely oriented spin and rewiring to a distant one of the same orientation. The distant spin may be an  $a$ -site with probability  $1/4$  and stability  $0.0$ ,  $d$ -site with probability  $1/2$ , stability  $0.5$  or an  $i$ -site with probability  $1/4$  and stability  $1.0$  (shown in Fig.3 by an elliptical boundary). Now due to the rewiring it changes respectively to a dormant site (say  $d''$ ) with stability  $1/3$  (Fig.3 Path B), a dormant site (say  $d'$ ) with stability  $2/3$  (Fig.3 Path C) or an  $i$  site (Fig.3 Path D) with no change in stability. On the other hand, the node from which a link is disconnected may be either an  $a$ -site with probability  $1/2$  or a  $d$ -site with probability  $1/2$ . The disconnected  $a$ -site transforms to an active site although with only one link (say  $a'$ ) (Fig.3 Path E) or a  $d$ -site to an  $i$  site with stability  $1.0$  (Fig.3 Path F). However the  $a'$  hardly remains stable and quickly transforms to an  $i$  site (Fig.3 Path E).

$$a \rightarrow d \quad (\text{Fig 3. Path A})$$

$$\frac{1}{4}a \rightarrow \frac{1}{4}d'' \quad (\text{Fig 3. Path B})$$

$$\frac{1}{2}d \rightarrow \frac{1}{2}d' \quad (\text{Fig 3. Path C})$$

$$\frac{1}{2}a \rightarrow \frac{1}{2}a' \rightarrow \frac{1}{2}i \quad (\text{Fig 3. Path D})$$

$$\frac{1}{2}d \rightarrow \frac{1}{2}i \quad (\text{Fig 3. Path E})$$

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$$\frac{7}{4}a \rightarrow \frac{1}{4}d'' + \frac{1}{2}d' + i \quad (\text{net conversion}) \quad (2)$$

Hence the net transformation leads to the reduction of  $7/4$   $a$  sites at each step and appearance of  $1/4$   $d''$ ,  $1/2$   $d'$  and an  $i$ - site. Again we can calculate approximately (ignoring the rigorous dynamics that really takes place) the increase in stability due to this transformation. It takes  $N/7$  steps for all the  $N/4$   $a$  sites to vanish. During this time, the new sites that appear are  $N/28$   $d''$ ,  $N/14$   $d'$  and  $N/7$   $i$  sites. So the increase in stability is  $(1/28 \times 1/3) + (1/14 \times 2/3) + (1/7 \times 1) = 0.202$ . As we begin with a stability of  $0.5$ , the final value we approach is  $0.702$  which is again  $0.02$  lower than that obtained from simulation. Now we can estimate the  $\langle s \rangle$  values for all other values of  $\phi$  from these two limiting values of stability enhancement. At  $\phi = 0.0$ ,  $\Delta\langle s \rangle = 0.3$  and at  $\phi = 1.0$ ,  $\Delta\langle s \rangle = 0.202$ . For any value of  $\phi$ ,  $\Delta\langle s \rangle = 0.3(1 - \phi) + 0.202\phi$ . The two branches for  $S < 0.5$  can also be explained from the instability of the  $d''$  sites when  $S > 1/3$ . In that case for  $\phi = 1.0$  the  $d''$  sites get immediately transformed to  $d'$  sites of stability  $2/3$  and thus  $\Delta\langle s \rangle = (1/28 \times 2/3) + (1/14 \times 2/3) + (1/7 \times 1) = 0.214$  and thus for any arbitrary  $\phi$ ,  $\Delta\langle s \rangle = 0.3(1 - \phi) + 0.214\phi$ . These two analytical lines have been shown in Fig.1(a) and are very close to those obtained from simulation.

The magnetisation shows a considerably high value  $\sim 1$  for values of  $S > 0.5$ , only at  $\phi = 0.0$  (Fig. 1b). However the value of magnetisation is very low otherwise.

The reason for the high value can be understood with a little insight. For  $\phi = 0$ , no rewiring occurs and the dynamics proceed only through flipping of spins.

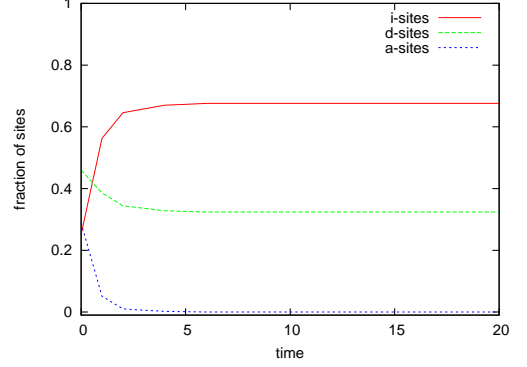


FIG. 4: The plot of time vs fraction of different sites, (i.e.,  $i$ ,  $d$  and  $a$  sites) for  $S = 0.3, \phi = 0.0$

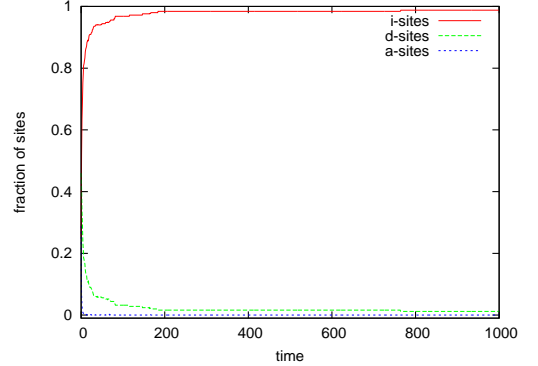


FIG. 5: The plot of  $time - fraction$  for different values of  $S = 0.6, \phi = 0.0$

According to the definition of the  $i$ ,  $d$  and  $a$  sites, the difference in dynamics at  $\phi = 0.0$  is due to the activity of the  $d$ -sites. When  $S < 0.5$ , most of the  $d$ -sites (whose stability factor is  $0.5$ ) remain dormant forever (Fig. 4). From our previous calculation, it can be approximated that  $4/5$  of the  $d$ -sites persist. Thus once all the  $a$ -sites are updated, the dynamics stop (even though a large fraction of  $d$ -sites remain intact as we begin with a random configuration). However some adjacent sites transform due to indirect effect. For example, during the update process, while an  $a$ -site converts to an  $i$ -site, an adjacent  $d$ -site transforms to an  $i$ -site. But as a whole, due to the presence of a large fraction of  $d$ -sites (in the equilibrium configuration), the magnetisation is very small.

The situation drastically changes for  $S > 0.5$ . For  $S > 0.5$ , the  $d$ -sites always flip and converts to another  $d$ -site (to fulfil the stability criterion) with a different configuration (i.e.  $++-$  becomes  $+-$  or  $--$  or  $++$  becomes  $-++$ ). So the domain walls perform a random

walk until they annihilate each other and all the sites become inactive asymptotically (Fig. 5). Obviously the  $a$ -sites also transform to  $i$ -sites. So as  $S$  exceeds the value of 0.5, all the spins become either up or down and thus the magnetisation reaches the value 1.0.

The plot of the fraction of free nodes left in the system,  $n_f$ , vs  $\phi$  is shown in Fig. 1c. The value of  $n_f$  increases with increasing probability of rewiring, but the nature of increase shows a marked difference for values of  $S \leq 0.5$  and  $S > 0.5$ .

Another interesting variation that we observed in this case was the variation of the value of  $\langle s \rangle$  with the fraction of up spins ( $\rho$ ).  $\rho = 0$  means all the spins are down and  $\rho = 1/2$  means equal number of up and down spins. This variation is measured for different values of  $S$  and  $\phi = 0$ , i.e., zero probability of rewiring (Fig. 6). It is observed that for values of  $S \leq 0.5$  the average stability per node  $\langle s \rangle$  decreases with  $\rho$  and reaches a minimum when  $\rho \approx 0.5$ . For values of  $S > 0.5$  however,  $\langle s \rangle$  remains almost constant ( $\sim 1$ ) with increasing  $\rho$ .

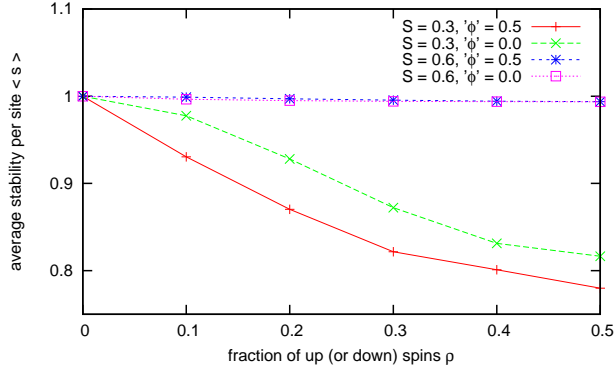


FIG. 6: Average stability per site versus fraction of up (or down) spins  $\rho$  at  $\phi = 0.0$  and  $\phi = 0.5$  for  $S = 0.3$  and  $S = 0.6$

### B. One dimensional chain with antiferromagnetic initialisation

This is a special case of the previously described lattice where the initial configuration is a one dimensional chain of nodes with alternate sites having spin  $+\sigma$  and  $-\sigma$ . As mentioned earlier,  $|\sigma| = 1$  and initially each node is connected with only nearest neighbours. We have separately studied this special case for two reasons: (i) to see how much the results vary with the previous one if we start off with a periodic array of spins and (ii) some results have already been derived exactly for the limiting case ( $\phi = 0.0$ ). Evidently,  $s_i = 0$  for all nodes as in this case we start off with  $l_i = 0$  and  $k_i = 2$  for all  $i$ . Therefore no matter how small a value of  $S$  we assign, a dynamics will take place to approach stability. Once the system reaches the equilibrium configuration we measure  $\langle s \rangle$ ,  $m$

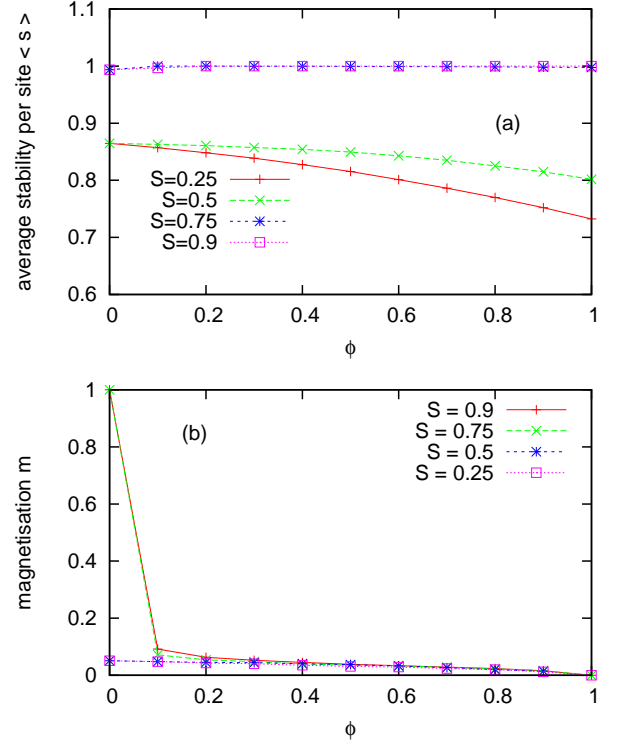


FIG. 7: (a) average stability per site  $\langle s \rangle$  vs  $\phi$ , (b) average magnetisation per site  $m$  vs  $\phi$

and  $n_f$ , as defined earlier

We show in Fig. 7(a) the  $\langle s \rangle$  vs  $\phi$  plots for different values of  $S$ . It is observed that we get three distinct branches for the various values of the preassigned stability factor  $S$ . For values of  $S \leq 0.5$ , we get two branches, and  $\langle s \rangle$  decreases with increasing value of  $\phi$ , however the two branches converge to the same value of  $\langle s \rangle$  at  $\phi = 0$ , namely  $\langle s \rangle_{(\phi=0)} \sim 0.86$ . When we start with an antiferromagnetic configuration of spins, all sites are  $a$ -sites, so that when  $S < 0.5$ , random updating leads to a final configuration that consists of domains of size greater than or equal to three. Updating of each  $a$ -site gives rise to a domain of odd number of sites and this process continues until all the  $a$ -sites vanish. It has been analytically proved that, in the steady state, the fraction of domain walls approaches a value of  $1/e^2$  [8, 9]. So fraction of  $d$ -sites reach the value  $2/e^2 = 0.2706$  and the remaining  $0.7294$  are  $i$ -sites. Since stability for a  $d$ -site is 0.5 and that for an  $i$ -site is 1.0, the stability factor approaches a value  $0.5 \times 0.2706 + 1.0 \times 0.7294 = 0.8647$ . On the other hand, when we start with a random initial configuration of spins, with  $S < 0.5$ , the fraction of  $a$ -sites starts from 0.5 and saturates at a value which is slightly higher (0.3243) than that obtained for antiferro initialisation. Consequently the fraction of  $i$ -sites decreases and thus the overall stability factor becomes lower (0.83785) as seen in Fig. 1(a).

The average magnetisation per site as we can see, is 1.0, only for  $\phi = 0.0$  and  $S > 0.5$ . Otherwise for any value of

$\phi$  and  $S$ , it is very low. So overall it is qualitatively same as for random initialisation.

### C. Effect of global magnetisation

In this subsection we intend to study the effect of global magnetisation on the system. Till now the spin flip, or rewiring with a distant node with same spin was dependent on the value of  $s_i$  and assigned  $\phi$ . However we observe significant changes if instead of local dependence, we introduce a global effect in the dynamics. Now a selected site will flip with probability  $1 - \phi$ , only if its spin does not match with the sign of the global magnetisation, i.e. the magnetisation of the system. In other words, it may so happen that even though a situation arises when flipping of the spin increases stability, the global magnetisation prohibits the system to gain that enhanced stability. Since global trends often appear as strong driving factors in societies, introduction of this global dependence makes our study more realistic. We observe the variation of the parameters  $\langle s \rangle$  (Fig. 8a) and  $m$  (Fig. 8b) with the rewiring probability  $\phi$  for different values of  $S$ . The most striking observation in this case is that both the average stability and magnetisation per site retains a considerably high value for a wide range of the rewiring parameter  $\phi$  for  $S > 0.5$ . So it can be inferred that following the global trend not only retains the high stability value but also brings about homogeneity to the system. Nevertheless the striking difference in the system behaviour for values of  $S \leq 0.5$  and  $S > 0.5$  is once again apparent from all the plots.

### D. Network with preferential attachment

It is a well known fact that a large number of real systems show the topology of a *Scale free network* [1]. In a nutshell, a scale free network is one in which the connection probability of a new node to an existing node is proportional to the degree (or number of links/neighbours) of the existing node, i.e.,

$$\Pi_i \sim k_i \quad (3)$$

at a given timestep. Here, the attachment is preferential instead of being random. For such networks, the degree distribution follows a power law, viz.  $P(k) \sim k^{-\gamma}$  and such networks are characterised by the existence of *hubs*, i.e., few nodes with very high concentration of links. Scale free networks form an extremely important genre of study for network theorists as several real world networks belong to this class. Keeping these in mind we next use a fully evolved scale free network as the substrate on which we place up or down spins on the nodes and carry out the dynamics mentioned earlier.

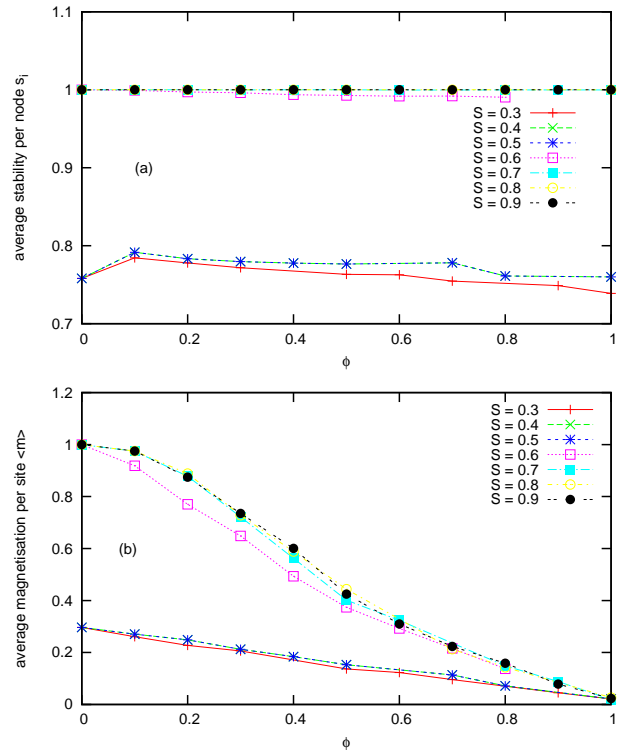


FIG. 8: (a) Average stability per node  $\langle s \rangle$  vs  $\phi$ , (b) Average magnetisation per site  $m$  vs  $\phi$

A very important modification to the Barabási-Albert type network is the one where the attachment probability has a nonlinear dependence on the degree [12], i.e.,

$$\Pi_i \sim k_i^\beta \quad (4)$$

In this case, it may be shown that the network is scale free, i.e., the degree distribution is a power law only for linear dependence, when  $\beta = 1.0$ . Such nonlinear modifications have been studied in details in [13, 14]. We made an investigation to find out any change in the magnetisation if the nonlinear degree dependence is introduced, when the system behaves as a small world instead of scale free. We found that indeed there is a drastic change in the magnetisation, which now showed considerably high values for  $\beta > 1.0$  with  $S = 0.9$  and  $\phi$  varying from 0 to 1.0 (Fig.10). This conclusively shows that the system parameters are dependent on the initial configuration of the substrate.

## IV. DISCUSSIONS

We have addressed here a simple model undergoing coevolution of node status and link structure. We have used the same update rule as [6] on different kinds of initial substrates. Spins randomly placed on nodes on a one dimensional lattice has been studied in some details where we have not only presented numerical results but

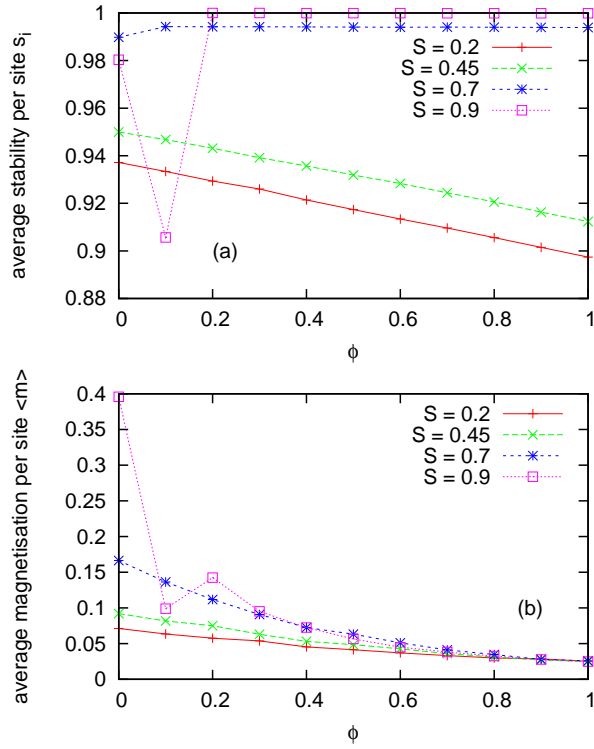


FIG. 9: (a) average stability per site  $\langle s \rangle$  vs  $\phi$ , (b) average magnetisation per site  $m$  vs  $\phi$

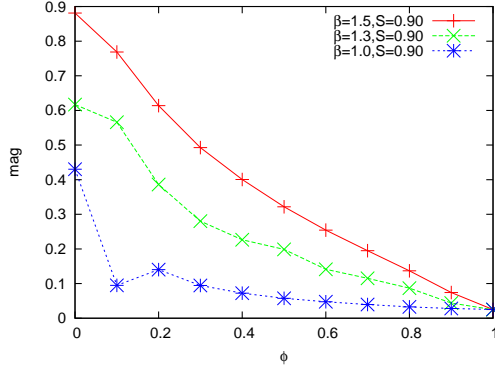


FIG. 10: Plot of magnetisation vs  $\phi$  for  $S = 0.9$  for three different values of network parameter  $\beta$ . Interestingly, for non linear dependence of the connection probability on degree of a node, considerably high values of magnetisation is obtained even for  $\phi > 0$

have also tried to put forward a theoretical explanation of the same. Simulations have also been made for the same lattice structure but when spins are placed in an antiferromagnetic fashion initially. The target stability can be thought of as a measure of the number of “like-minded” neighbours a particular agent should have in order to be called stable. Obviously, when the target stability is small, only the active sites ( $a$  sites) undergo dynamics and overall stability of the system shows a decrease. When the spin dynamics is considered and changes in  $s$  are calculated from corresponding rate equations, the theoretical and numerical results match considerably well albeit with slight difference in values. For the case where we have considered a network grown following the preferential attachment scheme, it is observed that although for scale free behaviour of the network, the variation of magnetisation with  $\phi$  does not show any significantly different behaviour, however as soon as we enter the non linear region, where according to [12, 14], scale free nature disappears and small world behaviour predominates and a “gel” formation takes place, we find the magnetisation to reach considerably high values even when  $\phi > 0.0$ . This conclusively shows that the system parameters depend on initial configuration of the agents.

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